Mock Exam. 2009: Selected Answers

7. (a) Subadditivity is a formal statement of the idea of a natural monopoly. For a single output natural monopoly, strict subadditivity requires

(1) $$C(Q) < C(q_1) + .. + C(q_k)$$

where $C(.)$ is total cost, $Q = \sum_{i=1}^{k} q_i$ is market output (where each firm's output is q_i), k is the number of firms and $k \geq 2$. Hence it is more efficient to have one firm than two or more firms producing market output.

(b) (i) In this case, $C(q_i) = 200 + 20q_i^2$ and so marginal cost is $MC_i = 40q_i$. Marginal cost therefore increases in output q_i. Under these cost conditions, whatever the number of firms, it would optimal for all firms be the same size. (If this was not so, costs could be reduced by reallocating outputs until marginal costs are equal).

Subadditivity requires

(2) $$C(Q) < C(q_1) + C(q_2)$$

Substituting in the total cost function gives:

$$200 + 20Q^2 < 200 + 20q_1^2 + 200 + 20q_2^2$$

Noting that $Q = q_1 + q_2$ gives

$$200 + 20(q_1 + q_2)^2 < 200 + 20q_1^2 + 200 + 20q_2^2$$

$$200 + 20q_1^2 + 20q_2^2 + 40q_1q_2 < 200 + 20q_1^2 + 200 + 20q_2^2$$

Cancelling common terms and noting that $q_1 = q_2$ gives

$$40q_1^2 < 200$$

and hence

$$q_1 < 2.24$$

The corresponding market output is $Q < 4.47$. Hence, as long as the actual market output is less than this, a natural monopoly exists.

(ii) In this case, $C(q_i) = 300 + 8q_i + 10q_i^2$ and so marginal cost is $MC_i = 8 + 20q_i$. Note that we have an extra term $8q_i$ in general form compared to the last question. However, putting $8Q$ on the left of the equation will cancel out with $8q_1$ and $8q_2$ on the right, so we are left with an equation similar to that in (i). By the same logic, there will be a natural monopoly if:

$$20q_1^2 < 300$$

and hence

$$q_1 < 3.87$$

The corresponding market output is $Q < 7.75$.

(c) Market demand is $p = 200 - 5q$ and total cost is $C(q) = 300 + 8q + 10q^2$. Hence, marginal cost is $MC = 8 + 20q$ and average cost is $AC = \frac{300}{q} + 8 + 10q$. Average cost is U-shaped and has a minimum where

$$\frac{dAC}{dq} = \frac{-300}{q^2} + 10 = 0$$

which implies

$$q = 5.48$$

Finally, the first best welfare outcome requires $p = MC$ and hence

$$200 - 5q = 8 + 20q$$

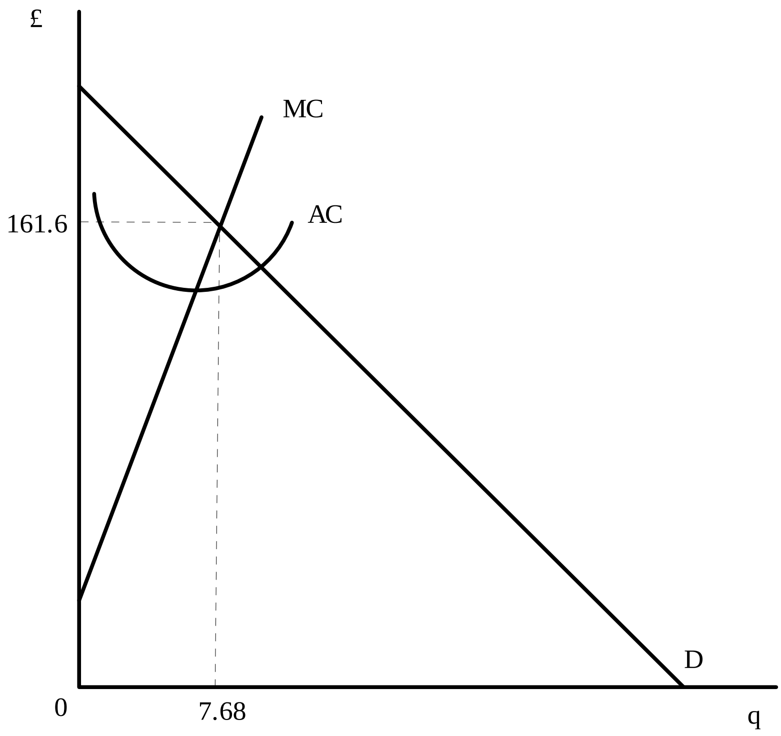
This implies

$$q = 7.68$$

and price is

$$p = 200 - 5(7.68) = 161.6$$

The diagram is on the next page. Since average cost has a minimum below the first best output and marginal cost is increasing, the firm will make a profit at this output as drawn.



8. (a) Ramsey pricing is a second best outcome designed to maximise welfare subject to a zero profit constraint. It overcomes the problem whereby, in a natural monopoly, first best pricing could lead to negative profits being earned. In single product natural monopoly, a single price *p* is set to meet the above condition e.g. the price of electricity. In the multiproduct case, a number of prices can be set. In the case where markets can be separated e.g. off-peak and peak electricity, setting more than one price will increase welfare (strictly cannot reduce it) compared to uniform pricing.

(b) The firm has a demand function $p = 100 - 2q$ and a total cost function $C = 800 + 4q$. It follows that marginal revenue is $MR = 100 - 4q$, marginal cost is $MC = 4$ and average cost is $AC = \frac{800}{q} + 4$.

<u>First Best Pricing</u> $p = MC$

$$100 - 2q = 4$$

and hence $\quad q = 48$

It follows that

$$p = 100 - 2(48) = 4$$

and

$$\pi = (4)(48) - 800 - 4(48) = -800$$

Second Best Ramsey Pricing $p = AC$

$$100 - 2q = \frac{800}{q} + 4$$

and hence

$$-2q^2 + 96q - 800 = 0$$

Using the quadratic formula, the positive root is

$$q = 37.27$$

It follows that

$$p = 100 - 2(37.27) = 25.47$$

and hence

$$\pi = (25.47)(37.27) - 800 - 4(37.27) \approx 0$$

Profit Maximisation $MR = MC$

$$100 - 4q = 4$$

and hence $q = 24$

It follows that

$$p = 100 - 2(24) = 52$$

and

$$\pi = (52)(24) - 800 - 4(24) = 352$$

The first best outcome involves price equal to marginal cost, 4 and output 48 but the firm makes a loss of 800. Second best Ramsey pricing sets a higher price, 25.47 and lower output 37.27 but the firm covers its costs. Under profit maximisation, the price would be higher, 52 and output is 24 (half the first best output). Clearly, profit maximisation involves higher prices compared to the other two outcomes.

(c) The deadweight loss can be calculated as

$$DWL = \frac{1}{2}\Delta p \Delta q$$

where Δp is the difference in prices and Δq is the difference in outputs between the first and second best outcome. There is no deadweight loss in first best pricing (ignoring here the way in which the firm's loss is paid for). In second best pricing,

$$DWL = \frac{1}{2}(25.47 - 4)(48 - 37.27) = 115.19$$

In this case, second best pricing involves a deadweight loss of 115.19. This is a potential gain which might accrue to either the firm or consumers if an alternative way can be found to cover the firm's losses under first best pricing. Subsidies would be possible but since these are raised from taxation (which also involves a deadweight loss (typically)) it is not clear that they represent a superior solution.