Handout 2 The Bertrand Model with Product Differentiation

In the Bertrand model it is assumed that each firm sets price to maximise profits. In what follows we provide a simple example of the case where products are differentiated.

Assume that there are two firms in the market, 1 and 2, producing different brands of soft drink. Firm 1 has a demand function $q_1 = 100 - 2p_1 + p_2$ and constant marginal cost, $c_1 = 10$ while firm 2 has a demand function $q_2 = 100 - 2p_2 + p_1$ and constant marginal cost, $c_2 = 15$. Assume, for simplicity, that there are no fixed costs.

Firm 1 has a profit function

(1) $$\pi_1 = (p_1 - 10)(100 - 2p_1 + p_2)$$

and the first order condition for profit maximisation is

(2) $$120 - 4p_1 + p_2 = 0.$$

Solving (2) for p_1 gives firm 1's reaction function

(3) $$p_1 = 30 + \frac{p_2}{4}$$

This shows firm 1's optimal price given the price of firm 2.

Firm 2 has a profit function

(4) $$\pi_2 = (p_2 - 15)(100 - 2p_2 + p_1)$$

and its first order condition is

(5) $$130 - 4p_2 + p_1 = 0.$$

Its reaction function is therefore

(6) $$p_2 = \frac{65}{2} + \frac{p_1}{4}$$

showing its optimal price given the price of firm 1.

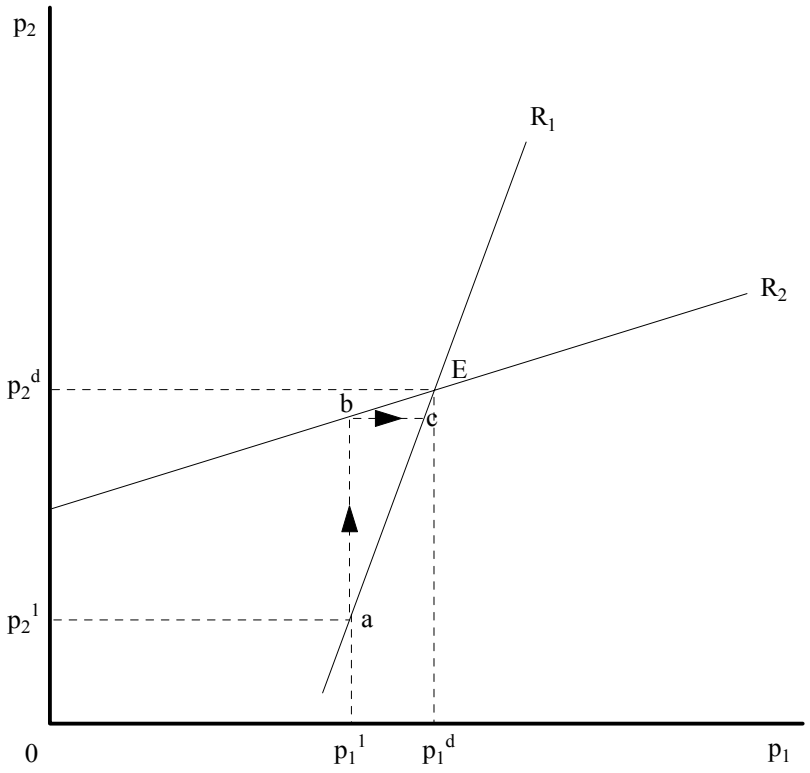
To solve the model substitute (6) into (3) to give

$$p_1 = 30 + \frac{1}{4}\left(\frac{65}{2} + \frac{p_1}{4}\right)$$

and so $p_1 = 40.7$.

Substitution back into equation (6) gives $p_2 = 42.7$. In addition, firm 1's output is $q_1 = 61.3$ and firm 2's output is $q_2 = 55.3$, while firm 1's profit is $\pi_1 = 1880.9$ and firm 2's profit is $\pi_2 = 1530.9$. Note that firm 1 has a lower price, higher output and higher profits than firm 2. Since firms have identical demand functions this arises because firm 1 has lower costs than firm 2.

The analysis can be illustrated using the reaction functions in equations (3) and (6). The curves are illustrated below. They have a positive slope (in contrast to the Cournot model in Handout 1).



If firm 2 initially sets a low price p_2^1, firm 1 will set price p_1^1 at point 'a' on its reaction curve R_1. It will then be optimal for firm 2 to increase its price to point 'b' on its reaction curve R_2, which induces firm 2 to increase its price, and so on. The Bertrand equilibrium is at point 'E' where the reaction curves intersect. Firms set prices $p_1 = 40.7$ and $p_2 = 42.7$.

Student Exercise. Assume that firm 1 has a demand function $q_1 = 500 - 2p_1 + p_2$ and everything else is as before. Derive the equilibrium firm prices, outputs and profits in this case. Explain why the results differ from the original model.