Optimum-Welfare and Maximum-Revenue Tariffs under Bertrand Duopoly

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Abstract

This note shows that the optimum-welfare tariff may exceed the maximum revenue tariff under Bertrand duopoly. This result is more likely the greater the competitiveness of the home firm relative to the competitiveness of the foreign firm, and the greater the degree of product substitutability.

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1. Introduction

In conventional trade theory, where a large country can use a tariff to improve its terms of trade, Johnson (1951/52) derived the well-known proposition that the maximum-revenue tariff exceeds the optimum-welfare tariff. When the tariff is set to maximise welfare, the marginal gain in tariff revenue equals the marginal loss of consumer surplus from the tariff. Therefore, tariff revenue must be increasing when welfare is maximised so the optimum-welfare tariff must be lower than the maximum-revenue tariff. In new trade theory, with imperfect competition, Brander and Spencer (1984) have shown that a tariff has a profit-shifting effect in addition to its effect on consumer surplus and tariff revenue, and Collie (1991) has shown that the optimum-welfare tariff may exceed the maximum-revenue tariff in a Cournot duopoly. With homogeneous products and linear demand, he shows that this will be the case unless the costs of the home firm relative to the costs of the foreign firm are sufficiently high. A similar result has been obtained by Larue and Gervais (2002) in a model where the domestic industry is a Cournot oligopoly competing with price-taking foreign firms.

Often results in the literature on trade policy under imperfect competition are criticised for not being robust especially to changes in the assumed market structure. Hence, an interesting question is whether the proposition that the optimum-welfare tariff may exceed the maximum-revenue tariff is altered if the market structure is Bertrand rather than Cournot duopoly. This note will address that question using the model employed by Clarke and Collie (2003) to show that there are always gains from intra-industry trade under Bertrand duopoly. This model has differentiated products so this note will also analyse how product substitutability affects the proposition that the optimum-welfare tariff may exceed the maximum-revenue tariff.

2. The Model

As in Clarke and Collie (2003), assume that there are two countries, a home and a foreign country and that each country has one firm that produces a differentiated good, with the home firm labelled as firm one and the foreign firm labelled as firm two. Consider the home market, where the two firms compete in a Bertrand duopoly. The home firm has constant marginal cost c_1, sets price p_1, and sells output y_1 while the foreign firm has constant marginal cost c_2, sets price p_2, and sells output y_2. The government in the home country imposes a specific import tariff of t per unit of imports from the foreign firm. It is assumed that there is a representative consumer in the home country with quasi-linear preferences that can be represented by a quadratic utility function, as in Vives (1985):

$$U(\mathbf{y},z)=\sum_{i=1}^{2}\alpha_i y_i-\frac{1}{2}\sum_{i=1}^{2}\beta_i y_i^2-\gamma y_1 y_2+z \qquad \alpha_i,\beta_i,\gamma>0;\quad \beta_i>\gamma \tag{1}$$

where z is consumption of a numeraire good and $0<\gamma^2/\beta_1\beta_2<1$ is a measure of the degree of product substitutability ranging from zero when the products are independent to one when they are perfect substitutes. It is straightforward to show that the utility function (1) yields the following inverse and direct demand functions:

$$\begin{aligned} p_i &= \alpha_i-\beta_i y_i-\gamma\, y_j \\ y_i &= \frac{1}{F}\left[\left(\alpha_i\beta_j-\alpha_j\gamma\right)-\beta_j\, p_i+\gamma\, p_j\right] \end{aligned} \qquad i,j=1,2 \quad i\neq j \tag{2}$$

where $F=\beta_1\beta_2-\gamma^2>0$. Assume that markets are segmented, then the profit functions of the home firm and the foreign firm from sales in the home country market are:

$$\pi_1=\left(p_1-c_1\right)y_1 \qquad\qquad \pi_2=\left(p_2-c_2-t\right)y_2 \tag{3}$$

Assuming an interior solution where both firms sell positive quantities in the home country market, it is straightforward to solve for the Bertrand equilibrium prices:

$$p_1 = c_1 + \frac{1}{H}\left[G(\alpha_1 - c_1) - \beta_1\gamma(\alpha_2 - c_2 - t)\right]$$
$$p_2 = c_2 + t + \frac{1}{H}\left[G(\alpha_2 - c_2 - t) - \beta_2\gamma(\alpha_1 - c_1)\right] \tag{4}$$

where $G = 2\beta_1\beta_2 - \gamma^2 > 0$ and $H = 4\beta_1\beta_2 - \gamma^2 > 0$. Substituting these prices into the demand functions (2) yields the Bertrand equilibrium sales of the home firm and the foreign firm:

$$y_1 = \frac{\beta_2}{FH}\left[G(\alpha_1 - c_1) - \beta_1\gamma(\alpha_2 - c_2 - t)\right]$$
$$y_2 = \frac{\beta_1}{FH}\left[G(\alpha_2 - c_2 - t) - \beta_2\gamma(\alpha_1 - c_1)\right] \tag{5}$$

It is useful to define the competitiveness of a firm as the difference between the maximum willingness to pay of the consumer for the product and the marginal cost of the firm. Thus, the competitiveness of the home firm is $(\alpha_1 - c_1) > 0$, and that of the foreign firm is $(\alpha_2 - c_2) > 0$. From (4) and (5), it can be seen that an increase in the tariff results in both firms increasing their prices, an increase in sales for the home firm and a decrease in sales for the foreign firm. These results can now be used to derive the maximum-revenue and the optimum-welfare tariffs.

The maximum-revenue tariff is the tariff rate that maximises the tariff revenue collected by the government in the home country. The tariff is t per unit of imports and the quantity of imports (sales of the foreign firm) is y_2 so tariff revenue is $R = ty_2$. Assuming an interior solution where the market is supplied by domestic production and foreign imports, the first-order condition for the maximisation of tariff revenue by the government in the home country is:

$$\frac{\partial R}{\partial t} = y_2 + t\frac{\partial y_2}{\partial t} = 0 \tag{6}$$

Thus, using (5), to solve for an explicit expression for the maximum-revenue tariff yields:

$$t^R = \frac{FH}{\beta_1 G} y_2 = \frac{1}{2}(\alpha_2 - c_2) - \frac{\beta_2 \gamma}{2G}(\alpha_1 - c_1) > 0 \tag{7}$$

Clearly, the maximum-revenue tariff must be positive if there are any imports, $y_2 > 0$, and it depends upon the competitiveness of the two firms. It is increasing in the competitiveness of the foreign firm, $(\alpha_2 - c_2)$, and decreasing in the competitiveness of the home firm, $(\alpha_1 - c_1)$.

The optimum-welfare tariff is the tariff that maximises the welfare of the home country, where the welfare of the home country is given by the sum of consumer surplus, the profits of the home firm and tariff revenue. Given the quasi-linear preferences in (1), consumer surplus is:

$$S = U - p_1 y_1 - p_2 y_2 - z = \frac{1}{2}\beta_1 y_1^2 + \frac{1}{2}\beta_2 y_2^2 + \gamma\, y_1 y_2 \tag{8}$$

Note that it can be shown that $dS = -y_1 dp_1 - y_2 dp_2$. Using (8), the welfare of the home country is:

$$W = S + \pi_1 + R = \frac{1}{2}\beta_1 y_1^2 + \frac{1}{2}\beta_2 y_2^2 + \gamma y_1 y_2 + (p_1 - c_1) y_1 + t y_2 \tag{9}$$

The government of the home country sets its tariff to maximise its welfare. Thus, assuming an interior solution where the home country market is supplied by domestic production and foreign imports, the first-order condition for welfare maximisation by the government of the home country is:

$$\frac{\partial W}{\partial t} = -y_2 \frac{\partial p_2}{\partial t} + (p_1 - c_1)\frac{\partial y_1}{\partial t} + t\frac{\partial y_2}{\partial t} + y_2 = 0 \tag{10}$$

The first term is the effect of the tariff on consumer surplus from imports, which is negative; the second term is the profit-shifting effect, which is positive; and the final two terms are the tariff revenue effect as in (6). When evaluated at the maximum-revenue tariff, the final two terms together are equal to zero so welfare will be increasing if the profit-shifting effect outweighs the effect on consumer surplus from imports. If this is the case then the optimum-welfare tariff will exceed the maximum-revenue tariff. Therefore, the greater the competitiveness of the home firm then the larger will be the profit-shifting effect and the more likely that the optimum-welfare tariff will exceed the maximum-revenue tariff.

Using (4) and (5) to solve (10) for an explicit solution for the optimum-welfare tariff yields:

$$t^W = \frac{F}{F+G}(\alpha_2 - c_2) > 0 \tag{11}$$

Clearly, the optimum-welfare tariff is positive and it is independent of the competitiveness of the home firm, just as in the Cournot duopoly case analysed by Collie (1991).

Since it is assumed that there is an interior solution, it is necessary to identify the parameter space where the sales of both firms will be positive quantities given that the government sets the optimum-welfare tariff. Substituting the optimum-welfare tariff (11) into the sales of the two firms (5) yields:

$$\begin{aligned} y_1^W &= \frac{\beta_2 G}{FH(F+G)}\left[(F+G)(\alpha_1 - c_1) - \beta_1\gamma(\alpha_2 - c_2)\right] \geq 0 \\ y_2^W &= \frac{\beta_1}{FH(F+G)}\left[G^2(\alpha_2 - c_2) - \beta_2\gamma(F+G)(\alpha_1 - c_1)\right] \geq 0 \end{aligned} \tag{12}$$

The $y_1^W = 0$ locus and the $y_2^W = 0$ locus are shown in figure 1, in terms of the competitiveness of the two firms, $(\alpha_1 - c_1)$ and $(\alpha_2 - c_2)$, so there will be an interior solution in the region between these two loci. Figure 1 can now be used to compare the optimum-welfare tariff and the maximum-revenue tariff. Subtracting (7) from (11) it can be shown that the optimum-welfare tariff will exceed the maximum-revenue tariff if:

$$\Delta t = t^W - t^R = \frac{1}{2}\left[\frac{\beta_2\gamma}{G}(\alpha_1 - c_1) - \frac{\beta_1\beta_2}{F+G}(\alpha_2 - c_2)\right] > 0 \tag{13}$$

By plotting the $\Delta t = 0$ locus in figure 1 it can be seen that it divides the region where there is an interior solution into two regions. Below the $\Delta t = 0$ locus, the optimum-welfare tariff exceeds the maximum-revenue tariff, and this occurs if the competitiveness of the home firm is sufficiently large relative to the competitiveness of the foreign firm. From (13), the optimum-welfare tariff will exceed the maximum-revenue tariff if:

$$J(\alpha_1 - c_1) > (\alpha_2 - c_2) \qquad \text{where} \qquad J \equiv \gamma\frac{F+G}{\beta_1 G} > 0 \tag{14}$$

The expression J is a function of the demand parameters (β_1, β_2, and γ) so depends upon the degree of product substitutability. Therefore, the optimum-welfare tariff is more likely to exceed the maximum-revenue tariff the larger is the expression J and the greater is the competitiveness of the home firm relative to the competitiveness of the foreign firm. This leads to the following proposition:

Proposition 1: *The optimum-welfare tariff exceeds the maximum-revenue tariff under Bertrand duopoly if the competitiveness of the home firm is sufficiently large relative to the competitiveness of the foreign firm,* $(\alpha_1 - c_1)/(\alpha_2 - c_2) > 1/J$.

As suggested, the greater the competitiveness of the home firm relative to the competitiveness of the foreign firm then the larger will be the profit-shifting effect and the more likely that the optimum-welfare tariff will exceed the maximum-revenue tariff.

Now consider how the degree of product substitutability affects the possibility that the optimum-welfare tariff exceeds the maximum-revenue tariff. Assume that the demand parameters facing the home firm and the foreign firm are identical ($\beta_1 = \beta_2 = \beta$) so the degree of product substitutability is: $\phi \equiv \gamma/\beta$, where $0 \le \phi < 1$. Then, the expression J in (14) can be written as a function of the degree of product substitutability:

$$J(\phi) = \gamma \frac{F+G}{\beta G} = \phi \frac{3-2\phi^2}{2-\phi^2} \tag{15}$$

This function is plotted in figure 2, which shows that $J(0)=0$ and that $J(1)=1$. It is increasing in the degree of product substitutability from $\phi = 0$ up to $\phi = \sqrt{9-\sqrt{33}}/2 \approx 0 \cdot 90$, and then decreases up to $\phi = 1$. Therefore, except when the products are very close substitutes, the greater the degree of product substitutability the more likely it is that the optimum-welfare tariff will exceed the maximum-revenue tariff. Note from figure 2 that $J(\phi)$ is greater than one for $\phi > \left(\sqrt{17}-1\right)/4 \approx 0 \cdot 78$. Then, in the symmetric case when the competitiveness of the home firm and the foreign firm are equal ($\alpha_1 - c_1 = \alpha_2 - c_2$), the optimum-welfare tariff will exceed the maximum-revenue tariff. This leads to the following proposition:

Proposition 2: *In the symmetric case, the optimum-welfare tariff exceeds the maximum-revenue tariff under Bertrand duopoly if the degree of product substitutability is sufficiently high,* $\phi > \left(\sqrt{17}-1\right)/4 \approx 0 \cdot 78$.

When the firms have the same cost and demand parameters, the optimum-welfare tariff will exceed the maximum-revenue tariff when the products are sufficiently close substitutes because the profit-shifting effect of the tariff will be large when the products are close substitutes.

3. Conclusions

In this note, it has been shown that the optimum-welfare tariff may exceed the maximum revenue tariff under Bertrand duopoly, just as in the case of Cournot duopoly analysed by Collie (1991) and Larue and Gervais (2002). This possibility arises due to the presence of imperfect competition and pure profits, but is robust to the form of market structure (Bertrand or Cournot duopoly). The optimum-welfare tariff may exceed the maximum-revenue tariff under imperfect competition because the tariff has a positive profit-shifting effect under Bertrand and Cournot duopoly, whereas there is no profit-shifting effect under perfect competition. The profit-shifting effect will be large when the relative competitiveness of the home firm is large and/or when the products of the two firms are close substitutes. Hence, this result is more likely the greater the competitiveness of the home firm relative to the competitiveness of the foreign firm and the greater the degree of product substitutability.

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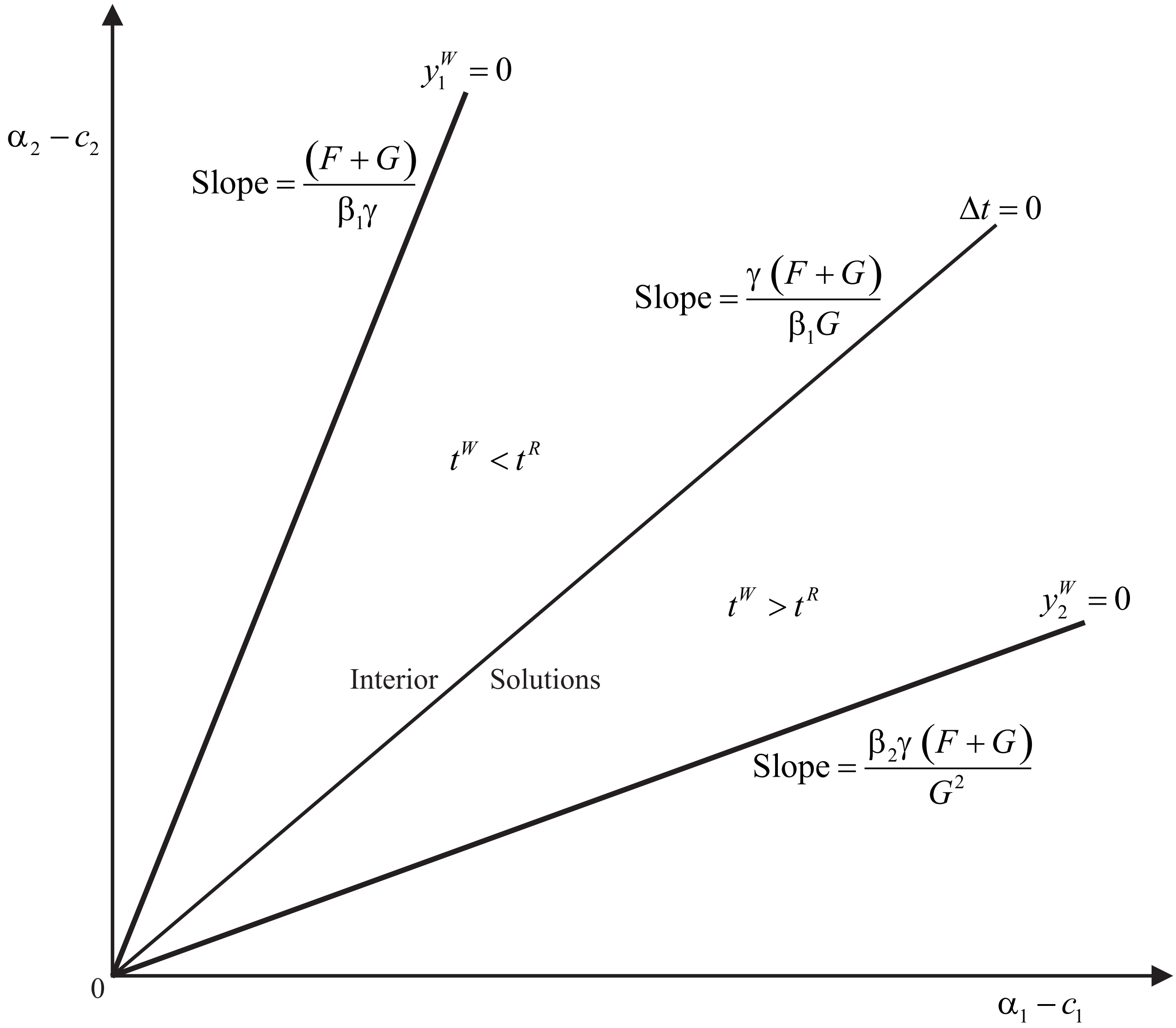

Figure 1: Optimum-Welfare and Maximum-Revenue Tariffs

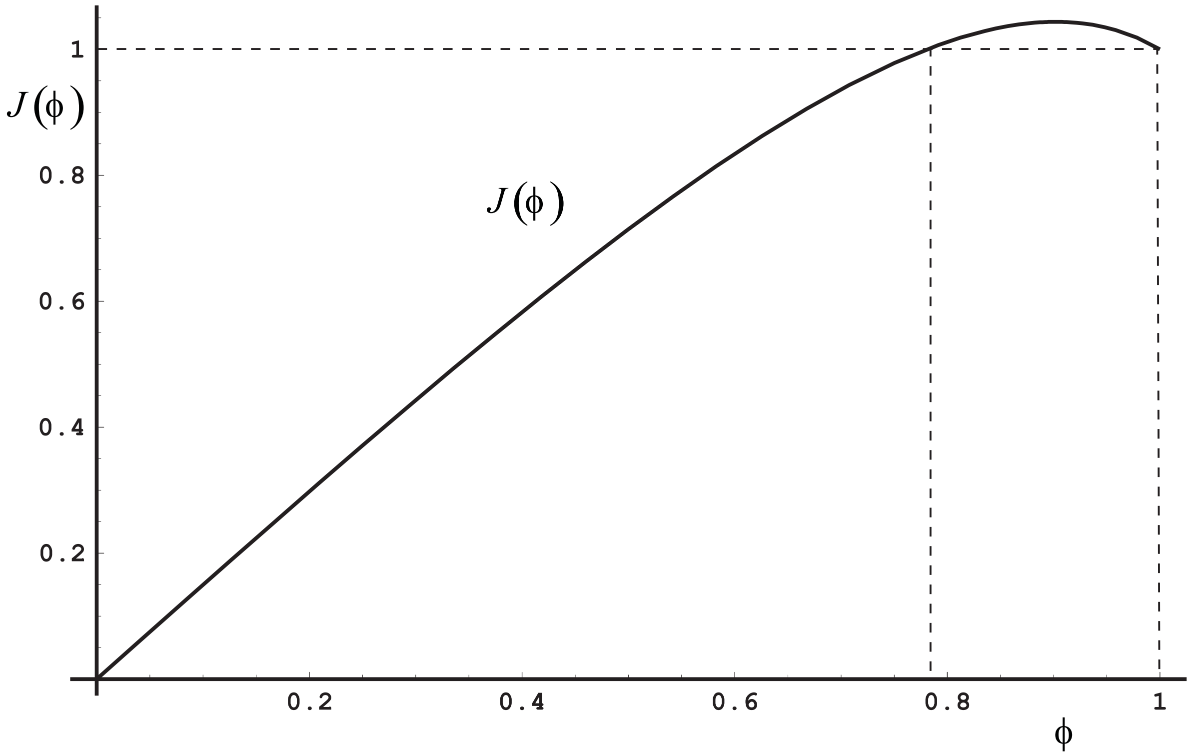

Figure 2: Product Substitutability